ACADÉMIE DE LA RÉPUBLIQUE SOCIALISTE DE ROUMANIE

REVUE ROUMAINE DE MATHÉMATIQUES PURES ET APPLIQUÉES

TOME XX, N° 3 1976

TIRAGE À PART

EDITURA ACADEMIEI REPUBLICII SOCIALISTE ROMÂNIA

JORDAN DECOMPOSITION AND LOCALLY ABSOLUTELY CONTINUOUS OPERATORS

BY

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By using the Jordan decomposition for vector measures it is shown that the class of all weakly compact operators defined on a space $C(S)$ and the class of all locally absolutely continuous operators are the same.

INTRODUCTION

The main result of this paper (see Theorem 2.5 below) establishes the identity between the class of all weakly compact operators defined on a space $C(S)$ and the class of all locally absolutely continuous operators (Definition 2.1 below) defined on the same space. Thus, the latter class appears as a class of summing (Corollary 2.6) or order continuous operators (Remark 2.8.) We pressent also a characterization of Banach lattices having order continuous topology (Corollary 2.9).

The crucial step in our proof is the establishment of the Jordan decomposition for vector valued measures. See Lemma [illegible] below. Surprisingly though the classical Jordan's result is elementary, the vector case involves deep results in the measure theory, namely the Dunford-Pettis theorems in [4].

For the convenience of the reader we summarized in section 1 of the present paper some basic facts about the relation between measures and operators.

1. REVIEW OF THE MEASURE THEORY

For a more detailed account of the content of the present section we refer to [7] and [13].

For $\mathscr{C}$ a Boolean ring (see [9] ch. 2 exercise 11 or [13] Definition 1.1) and Y a sequentially complete locally convex Hausdorff space denote by $\mathrm{Mes}_Y(\mathscr{C})$ the vector space of all additive measures $m : \mathscr{C} \to Y$ which are locally bounded, i.e.

$$\sup_{\substack{B \subset A \\ B \in \mathscr{C}}} p(m(B)) < \infty$$

for each $A \in \mathscr{C}$ and each continuous semi-norm p on Y.

REV. ROUM. MATH. PURES ET APPL., TOME XXI, Nº 3, P 343–352, BUCAREST, 1976

If Y is a locally convex vector lattice (i.e. the topology of Y is generated by semi-norms p such that $|x| \leqslant |y|$ implies $p(x) \leqslant p(y)$) then every positive measure $m : \mathscr{C} \to Y$ belongs to $\mathrm{Mes}_Y(\mathscr{C})$. Therefore, a Jordan type theorem can be formulated only for locally bounded measures.

In the next the elements of $\mathrm{Mes}_Y(\mathscr{C})$ will be regarded as continuous linear operators on the space of all totally $\mathscr{C}$ — measurable functions. The general case was treated in [13]. We consider here the case $\mathscr{C}$ a ring of subsets of an abstract set T, originally due to Dinculeanu [3]. For every $A \in \mathscr{C}$ denote by $\mathscr{M}_A(\mathscr{C})$ the completion in the sup norm of the space of all $\mathscr{C}$ — step (i.e. simple) real functions defined on T and vanishing outside A. Define :

$$\mathscr{M}(\mathscr{C}) = \varinjlim_{A \in \mathscr{C}} \mathscr{M}_A(\mathscr{C})$$

in the category of all locally convex Hausdorff spaces. There is defined a natural algebraic isomorphism :

$$\Phi_{\mathscr{C},Y} : \mathrm{Mes}_Y(\mathscr{C}) \xrightarrow{\sim} \mathscr{L}(\mathscr{M}(\mathscr{C}), Y)$$

given by :

$$\Phi_{\mathscr{C},Y}(m)(\chi_A) = m(A)$$

where χ_A denotes the characteristic function of A. Furthermore, if Y is a locally convex lattice this isomorphism preserves the order.

A useful improvement of this isomorphism was obtained by J. Hoffmann-Jørgensen [7] :

1.1 THEOREM. *Let $\mathscr{C}$ be a Boolean algebra. In order that the image by $\Phi_{\mathscr{C},Y}$ of a measure $m \in \mathrm{Mes}_Y(\mathscr{C})$ be a weakly compact operator it is necessary and sufficient that m be a strongly additive measure, i.e. for every sequence $\{A_n\}_n$ of disjoint elements, the sequence $\{m(A_n)\}_n$ converges to $0 \in Y$.*

For $\mathscr{C}$ a Boolean algebra the Banach space $\mathscr{M}(\mathscr{C})$ is isometric to a space $C(S)$ where the compact Hausdorff space S is obtained by using the Stone representation theorem for Boolean algebras. Thus, Theorem 1.1 above is an immediate consequence of the Kluvanek's result concerning the extension of vector measures and of the following result in [1] (see also [6]) :

1.2 THEOREM. *Let S be a compact Hausdorff space and let $\mathscr{B}(S)$ be the Borel σ-ring associated to S. Then there exists a natural algebraic isomorphism between the vector space of all σ-additive measures $m : \mathscr{B}(S) \to Y$ and the vector space of all weakly compact operators $U \in \mathscr{L}$ $(C(S), Y)$. Moreover, m and U are related by :*

$$U(f) = \int f \, \mathrm{d}m$$

for each $f \in C(S)$.

The isomorphism $\psi_{S,Y}$ above is related to the isomorphism $\Phi\mathscr{B}(S)_Y$ as follows: for every σ-additive measure $m : \mathscr{B}(S) \to Y$, $\psi_{S,Y}(m)$ is the restriction of $\Phi\mathscr{B}_Y(m)$ to $C(S)$.

Form the result above it follows (see [6] that an operator $U \in \mathscr{L}(C(S), Y)$ is weakly compact if, and only if, U maps weakly Cauchy sequences into convergent sequences or equivalently, if U^{**} maps every open F_σ — subset of S into an element of Y. In the next section we shall present another characterization by using the notion of absolute continuity. Our result is seen to be intimately related to the following in the measure theory: Let m be a measure defined on a σ-algebra $\mathscr{T}$ and taking values in a Banach space X. Then m is σ-additive if, and only if, there exists a σ-additive positive measure $\mu : \mathscr{T} \to \mathbb{R}_+$ such that m be absolutely continuous with respect to μ. See [1] or the footnote on the page 146 in [6].

2. LOCALLY ABSOLUTELY CONTINUOUS OPERATORS ON C(S)

Let Z be a locally convex lattice, X a Banach space, $T \in \mathscr{L}(Z,X)$ and $z^* \in Z^*$, $z^* \geqslant 0$.

2.1. DEFINITION. *T is said to be locally absolutely continuous with respect to z^* (i. e. $T \ll z^*$) if for each $z \in Z$, $z > 0$ and each $\varepsilon > 0$ there exists $\delta = \delta(z, \varepsilon) > 0$ such that:*

$$|y| \leqslant z,\ z^*(|y|) < \delta \text{ implies } \|Ty\| \leqslant \varepsilon$$

An operator $T \in \mathscr{L}(Z, X)$ is said to be locally absolutely continuous if there exists $z^* \in Z^*$, $z^* > 0$ such that $T \ll z^*$. We shall denote by $AC(Z,X)$ the vector subspace of all locally absolutely continuous operators $T \in \mathscr{L}(Z, X)$.

The notion of locally absolutely continuous operator was introduced in [13] in connection with the Bourbaki's version for the Lebesgue-Nikodym theorem.

For $Z = C(S)$, S a compact Hausdorff space, each $T \in AC(Z, X)$ maps weakly summable sequences into summable sequences, which implies that T is weakly compact. The main result of this section asserts that the converse is also true. We need several lemmas. The first one establishes the Jordan decomposition for an important class of vector measures:

2.2. LEMMA . *Let S, S' be two compact Hausdorff spaces and let $T \in \mathscr{L}(C(S), C(S'))$ a (weakly) compact operator. Then there are two (weakly) compact positive operators $T_1, T_2 \in \mathscr{L}(C(S), C(S')^{**})$ such that $T = T_1 - T_2$.*

If in addition S' is supposed to be a Stonean space then there exists a positive projection $P : C(S')^{**} \to C(S')$ (use the Hahn-Banach theorem) and thus we can choose $T_1, T_2 \in \mathscr{L}(C(S), C(S'))$.

Proof. It suffices to prove that every (weakly) compact operator $S \in \mathscr{L}(L^1(\mu), L^1(\nu))$ is order majorized by a (weakly) compact positive one (use the natural duality between the AM and AL. Or this follows from the Dunford-Pettis-Phillips theorem concerning the integral representation of the (weakly) compact operators defined on a space $L^1(\mu)$ (see

[5] Thms. 9. 4. 7 and 9.4.8) and from the fact that if A is a relatively (weakly) compact subset of $L^1(\nu)$ then

$$|A| = \{|f|\,;\, f \in A\}$$

is also relatively (weakly) compact. The case "weakly compact" follows immediately form the Dunford-Pettis characterization for the weakly compact subsets of a space $L^1(\nu)$. See [5] Theorem 4.21.2. The "case compact" is elementary.

2.3. Remark . The version "compact" was first established by Krengel [10] by using a different (elementary) method which cannot be adapted to the weakly compact case.

In the next section we shall obtain the Jordan decomposition in a more general situation.

The following result can be deduced from [13] Théorème 3.4. We present here a direct proof.

2.4. Lemma . *Let X be an ordered Banach space (i.e. $0 \leqslant x \leqslant y$ implies $\|x\| \leqslant \|y\|$) and let $T \in \mathscr{L}(C(S), X)$ a weakly compact positive operator. Then $T \in AC(C(S), X)$.*

Proof. Since T is weakly compact there exists a positive measure μ, on S, such that :

$$f \in \mathscr{M}(\mathscr{B}(S)), \int |f| \, d\mu = 0 \text{ implies } T^{**}(f) = 0$$

See [1] or the footnote on the page 146 in [6]. We shall show that $T \ll \mu$. Suppose that the contrary is true. Then, there exist an $\varepsilon_0 > 0$ and a sequence $f_n \in C(S)$ such that :

$$\text{i.} \quad 0 \leqslant f_n \leqslant 1$$

$$\text{ii.} \int f_n \, d\mu \leqslant \frac{1}{n+1}$$

$$\text{iii.} \quad \| T(f_n) \| \geqslant \varepsilon_0$$

Consider the following elements of $\mathscr{M}(\mathscr{B}(S))$:

$$g_n = \sup \{f_k \;;\; k \geqslant n\}$$

$$g = \inf \{g_n \;;\; n \geqslant 1\}$$

As usual we consider on $\mathscr{M}(\mathscr{B}(S))$ the pointwise order. Since $\int g \, d\mu = 0$ we have $T^{**}g = 0$ and from the Beppo-Levi theorem in the measure theory it follows that $\{T^{**} g_n\}_n$ converges weakly to $T^{**}(g)$. Here we consider T^{**} with values in X. On the other hand, $\{T^{**} g_n\}_n$ is a decreasing sequence of positive elements. Then, a well-known result due to Dini (see [17] ch. V 4.3) asserts that $\{T^{**}g_n\}_n$ converges to 0 in the norm topology of X, in contradiction to the fact that:

$$\| T^{**}g_n \| \geqslant \| T^{**}f_n \| \geqslant \varepsilon_0$$

q.e.d.

In order to formulate our main result we need a definition. Let Z be a locally convex lattice and consider the vector space $l^1[Z]$ (respectively $l^1(Z)$) of all weakly summable (resp. summable) sequences of elements of Z. See Pietsch [16] for details. We shall denote by $l^1_o\,[Z]$ the vector subspace of all $\{z_n\}_n \in l^1[Z]$ such that for a suitable $z > 0$ we have:

$$\sum_{n\in F} |z_n| \leqslant z$$

for every finite subset F of $\mathbb{N}$.

2.5. THEOREM. *For $T \in \mathscr{L}(C(S), X)$ the following statements are equivalent:*

(a) *T is weakly compact*
(b) *T is locally absolutely continuous*
(c) *$T\,(l^1_o[C(S)]) \subset l^1(X)$*

One can show that $l^1_o\,[C(S)] = l^1[C(S)]$ and in this form the equivalency $(a) \Leftrightarrow (c)$ was first remarked by Pelczynski [15].

Proof. $(a) \Rightarrow (b)$. Let K be the unit ball of X^* endowed with the weak*-topology and denote by $i_K : X \to C(K)^{**}$ the canonical mapping. Then $i_K \cdot T$ is weakly compact and Lemma 2.2 above reduces our problem to the case of a weakly compact positive operator of $\mathscr{L}(C(S), C(K)^{**})$. It remains to apply Lemma 2.4 above, q.e.d.

$(a) \Rightarrow (c)$ follows easily from the Orlicz-Pettis theorem concerning the unconditional weakly convergent sequences

$(b) \Rightarrow (a)$ and $(c) \Rightarrow (a)$ follow from the fact that T^{**} maps every open F_σ-subset of S into an element of X. Apply Theorem 6 in [6].

2.6. COROLLARY. *An operator $T \in \mathscr{L}(Z, X)$ is locally absolutely continuous if, and only if, T satisfies the following two conditions:*

(*i*) *There exists $z^* \in Z^*$, $z^* > 0$, such that*

$z \in M(Z)$, $z > 0$, $z^*(z) = 0$ *implies* $T^{**}(z) = 0$

(*ii*) $T(l^1_o\,[Z]) \subset X$

Here $M(Z)$ denotes the monotone hull of Z in Z^{**}, i.e. if $\{z_n\}_n$ is a decreasing sequence of elements of $M(Z)_+$ then its $\sigma(Z^{**}, Z^*)$—limit is so in $M(Z)$. *For Z a space $C(S)$ we can use $M(B(S))$ instead of $M(Z)$ and in this case (i) and (ii) imply that $T \ll z^*$.*

Proof. The necessity is clear. The sufficiency. Let $z \in Z$, $z > 0$. Consider the following normed space:

$$Z_z = \{y \in Z\,;\, |y| \leqslant \lambda z \text{ for a suitable } \lambda > 0\} \text{ normed by :}$$

$$\|y\|_z = \inf\,\{\lambda > 0\,;\, |y| \leqslant \lambda z\}$$

Then Z_z is isometric and order isomorphic to a space $C(S)$ for S a suitable compact Hausdorff space. Denote by $i_z : Z_z \to Z$ the canonical inclusion. By virtue of (*ii*) it follows (see $c \Rightarrow a$ in Theorem 2.5 above) that $T_0 i_z$ is weakly compact. As in the proof of Lemma 2.4 we can conclude (see Remark 2.10 bellow) that $T_0 i_z \ll z^k_\vee z \geqslant 0$.

2.7 Remark. Let $T \in \mathscr{L}(Z, X)$ such that $T^{**}(M(Z))$ be separable. Then T satisfies (i) for a suitable $z^* \in Z^{***}$, $z^* > 0$. See [13] Lemma 4.4 for the proof.

2.8 Remark. The condition 2.6 (ii) above is equivalent to each of the following assertions:

(*ii*)′ T maps every order interval into a conditionally weakly compact subset of X.

(*ii*)″ If $\{z_n\}_n$ is a decreasing sequence of positive elements of Z, then $\{Tz_n\}_n$ is a convergent sequence.

The proof is an immediate consequence of the remarks following theorem 1.2 above. Use the spaces Z_z.

Particularly from the result above we can deduce the following characterization of Banach lattices having order continuous topology:

2.9. Corollary. *For Z an order σ-complete Banach lattice the following statements are equivalent*:

(*1*) $z_n \downarrow 0$ (*in order*) *implies* $\|z_n\| \to 0$

(*2*) *Each order interval of Z is relatively weakly compact*

(*3*) $l^1_o[Z] \subset l^1(Z)$

(*4*) *For every compact Hausdorff space S we have*:
$T \in \mathscr{L}(C(S), Z)$, $T \geqslant O$ *implies* T = *weakly compact*

See also [14].

2.10. Remark. If X is an ordered Banach space and $U, V \in \mathscr{L}(C(S), X)$ we have:

$O \leqslant U \leqslant V$, V = *weakly compact implies* U = *weakly compact*

This follows from Theorem 2.5.

Particularly if S and S' are two compact Husdorff spaces, S' being supposed in addition Stonean, then the weakly compact operators $T \in \mathscr{L}(C(S), C(S'))$ constitute a lattice.

3. JORDAN DECOMPOSITION FOR VECTOR MEASURES

An important result in the measure theory asserts that every σ-additive measure defined on a Boolean σ-algebra is the difference of two σ-additive positive measures. In literature this result is known as Jordan decomposition theorem. In the next we shall consider σ-additive measures taking values in a Banach lattice Z. In order that the Jordan decomposition hold for every such measures, several restrictions must be imposed on Z, e.g. by considering the σ-additive measures on $\mathscr{P}(\mathbb{N})$, the σ-algebra of all subsets of $\mathbb{N}$, the following condition is necessary :

$$(aM) \qquad \{z_n\}_n \in l^1(Z) \Rightarrow \{|z_n|\}_n \in l^1(Z)$$

In fact, every element of $l^1(Z)$ can be interpreted as a σ-additive measure $m : \mathscr{P}(\mathbb{N}) \to Z$.

It was proved by D.I. Cartwright and H. P. Lotz (Some characterizations of AM and AL spaces, to appear) that aM implies Z is lattice isomorphic to an AM space in the sense of Kakutani. However, not every measure having as range an AM space is decomposable.

3.1. PROPOSITION. *For $\mathscr{C}$ a Boolean algebra and $m \in \mathrm{Mes}_{c_0}(\mathscr{C})$ the following assertions are equivalent* :

i) $\Phi_{\mathscr{C},c_0}(m)$ *is a compact operator*

ii) *There are two positive measures* m_1, $m_2 \in \mathrm{Mes}_{c_0}(\mathscr{C})$ *such that* $m=m_1-m_2$

Proof. i) $\Rightarrow$ ii). If $\Phi_{\mathscr{C},c_0}(m)$ is a compact operator then the result of Krengel in [10] (see also A. Peressini : Ordered topological vector spaces, page 179) implies the existence of two compact positive operators U, $V \in \mathscr{L}(\mathscr{M}(\mathscr{C}), c_0)$ such that $\Phi_{\mathscr{C},c_0}(m) = U - V$. Then ii) is satisfied for :

$$m_1 = \Phi_{\mathscr{C},c_0}^{-1}(U)$$

and

$$m_2 = \Phi_{\mathscr{C},c_0}^{-1}(V)$$

ii) $\Rightarrow$ i). If $m \in \mathrm{Mes}_{c_0}(\mathscr{C})$ is a positive measure then $\Phi_{\mathscr{C},c_0}$ (m) is a positive operator which maps the unit ball of $\mathscr{M}(\mathscr{C})$ into an order interval of c_0. Or $\mathscr{M}(\mathscr{C})$ is lattice isometric to a $C(S)$ space and each order interval of c_0 is relatively compact, q.e.d.

Particularly the above result implies that the vector measure $m(A) = \left\{\int_A \cos nt \, \mathrm{d}t\right\}_n$ from the Borel subsets of $[0, 2\pi]$ with values in c_0 is not decomposable as a measure having values in c_0. However, there exists a decomposition in $\mathrm{Mes}_{1\infty}(\mathscr{C})$ given by :

$$m_+(A) = \left\{\int_A (\cos nt)^+ \mathrm{d}t\right\}_n$$

$$m_-(A) = \left\{\int_A (\cos nt)^- \mathrm{d}t\right\}_n$$

In the next we shall study a slightly different notion of decomposability. Our results are in connection with Theorem 3.8 in [13].

For X an arbitrary Banach space we shall denote by $S_1(X^*)$ the unit ball of X^* endowed with the weak*-topology. A closed subset $K \subset S_1(X^*)$ is called an essential subset if :

$$\| x \| = \sup_{x^* \in K} |< x, x^* >|$$

for each $x \in X$. For $X = C(S)$ we can identify S as an essential subset.

Fix an essential subset K of $S_1(X^*)$. There is defined a natural mapping :

$$i_K : X \longrightarrow C(K)^{**}$$

which is an isometry. The following result is an esy improvement of Lemma 2.2 above.

3.2. PROPOSITION. *Let $T \in \mathscr{L}(C(S), X)$ a weakly compact operator Then $i_K \circ T$ is the difference of two weakly compact positive operators belonging to $\mathscr{L}(C(S), C(K)^{**})$.*

Particularly for $X = \mathbb{R}$ we can choose $K = \{1\}$ and in this case $C(K) \simeq \mathbb{R}$. Therefore, every real Radon measure is the difference of two positive Radon measures. This result is equivalent to the classical Jordan's result.

From 1.1 and 3.2 it follows immediately :

3.3. PROPOSITION. *Let X, K, i_K be as above and let $\mathscr{T}$ be a Boolean algebra. If $m : \mathscr{T} \to X$ is a strongly additive measure then $i_K \circ m = m_1 - m_2$ for $m_1, m_2 : \mathscr{T} \to C(K)^{**}$ two strongly additive positive measures.*

By virtue of Remark 2.10 above, this result remains true for locally strongly additive measures defined on Boolean rings. In the next we are interested to formulate a Jordan theorem for σ-additive measures. Our result is in connection with the existence of the control measures.

Let $\mathscr{S}$ be a Boolean δ-ring, X a Banach space and $\mu : \mathscr{S} \to \mathbb{R}_+$ a σ-aditive positive measure. Denote by $\mathrm{Mes}_X(\mathscr{S}, \mu)$ the vector subspace of all $m \in \mathrm{Mes}_X(\mathscr{S})$ such that m is locally absolutely continuous with respect to μ, i.e. $m \ll \mu$ (see [13] Definition 1.9). Analogously for Z a locally convex lattice and $z^* \in Z^*$, $z^* > 0$ denote by $AC_{z^*}(Z, X)$ the vector subspace of all $T \in \mathscr{L}(Z, X)$ such that $T \ll z^*$.

The following result motivates the use of the term of local absolute continuity in 2.1 above :

3.4. THEOREM. *The isomorphism $\Phi_{\mathscr{S},X}$ induces naturally the following algebraic isomorphism :*

$$\mathrm{Mes}_X(\mathscr{S}, \mu) \xrightarrow{\sim} AC_{\Phi_{\mathscr{S},\mathbb{R}}(\mu)}(\mathscr{M}(\mathscr{S}), X)$$

Proof. It is no harm to assume that $\mathscr{S}$ is a Boolean σ-algebra. If we denote by S the spectrum of $\mathscr{S}$, then $\mathscr{S}$ is isomorphic to the Boolean algebra $\mathscr{S}'$ of all clopen subsets of S and $\mathscr{M}(\mathscr{S})$ is equivalent (as Banach lattice) to $C(S)$. Let $m \in \mathrm{Mes}_X(\mathscr{S}, \mu)$. Considered as measures defined on

$\mathcal{S}'$, m and μ are both strongly additive and σ-additive and, thus (see [18]), they have a unique σ-additive extension to the σ-algebra generated by $\mathcal{S}'$. If we denote these extensions by m' and, respectively, μ' then $m' \ll \mu'$ and our result is a consequence of Corollary 2.6 above

3.5. THEOREM. *Let S be a compact space, let μ be a positive Radon measure on S and let $T \in \mathcal{L}(C(S), X)$ such that $T \ll \mu$. Then $|i_K \circ T| \ll \mu$.*

Proof. Since $C(K)^{**}$ is lattice isometric to a space $C(H)$ for H a compact Stonean space then there exists the modulus of $i_K \circ T$ in $\mathcal{L}(C(S), C(K)^{**})$. See A. Peressini : Ordered topological vector spaces, page 22.

By combining Theorem 2.5, Lemma 2.2. and Remark 2.10 above we obtain that $|i_K \circ T|$ is weakly compact.

Let $f \in \mathcal{M}(\mathcal{B}(S))$, $f \geq 0$, such that $\int f \, d\mu = 0$. Since $T \ll \mu$ we have

$$0 \leq g \leq f,\ g \in \mathcal{M}(\mathcal{B}(S)) \text{ implies } T^{**}(g) = 0$$

On the other hand :

$|i_K \circ T|^{**}(f) = \sup\{|i_K \circ T(g)|;\ |g| \leq f,\ g \in \mathcal{M}(\mathcal{B}(S))\}$ and thus $|i_K \circ T|^{**}(f) = 0$. Therefore, we can continue as in the proof of Lemma 2.4 above.

3.6 COROLLARY. *Let $\mathcal{T}$ be a σ-algebra and let $m : \mathcal{T} \to X$ a σ-additive measure. Then, $i_K \circ m = m_1 - m_2$ for $m_1, m_2 : \mathcal{T} \to C(K)^{**}$ two σ-additive positive measures.*

Hint. It was remarked in [1] that there exists a σ-additive positive $\mu : \mathcal{T} \to R_+$ such that $m \ll \mu$.

Received November 30, 1973

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